Fractal University

Fractal Geometry Curriculum

Based upon six strands, major fractal concepts have been identified at the beginner, intermediate and advanced levels. These concepts can be taught in math and science classes, grades 5 to 12, and are the basis of our *Fractal Keys* tours.

1. Fractals are **self-similar**.

2. Fractals are formed by a **repetitive process** in which each repetition builds on the prior result.

3. Fractals look the same at any **scale**.

4. Fractals have unique **dimensions** (roughness) that can be mathematically described.

5. **Computer technology** has enabled centuries of mathematical research to be understood and expanded to form the discipline of fractal geometry.

6. Fractal patterns are used to **model nature** in creating art, music, architecture and new processes, tools, and products that benefit society.

Fractal Geometry Curriculum

Self-Similar

1. Fractals are self-similar

Beginner	Intermediate	Advanced
1.11 Similar objects have the same shape, but not necessarily the same size.	1.21 A statistical self-similar object is composed of parts approximately shaped like itself.	1.31 Fractals are the patterns in chaos.
1.12 A self-similar object is composed only of parts shaped like itself.	1.22 A statistical fractal is approximately self-similar.	1.32 When randomly generated points are pulled into a pattern toward a given point, the point is called a "strange attractor".
1.13 Mathematical fractals are self-similar. Natural fractals are approximately self-similar.	1.23 Statistical fractals occur throughout the natural world. They may be called natural or random fractals.	1.33 Fractals are the most commonly occurring "strange attractors" in nature and frequently occur at the boundaries between two different zones or basins of attraction.
1.14 The formation of self-similar patterns requires geometric transformations: translation, reflection, dilation and rotation.	1.24 In mathematics, a fractal sequence is one that contains itself as a proper subsequence. An example is 1, 2, 1, 2, 3, 1, 2, 3, 4, 1, 2, 3, 4, 5, 1, 2, 3, 4, 5, 6, ...	1.34 Relationships exist among Pascal's triangle, Sierpinski's triangle and Cellular Automata.
1.15 Fractals have patterns within patterns (in contrast to growing and repeating patterns).	1.25 There are several methods to test a numerical sequence to determine if it is a fractal sequence including upper and lower cuts.	
1.16 Self-similarity is found in many living organisms.		

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Repetitive Process

2. Fractals are formed by a repetitive process in which each repetition builds on the prior result.

Beginner	Intermediate	Advanced
2.11 To **iterate** means to repeat the same step or procedure. 2.12 **Recursion** is the process of repeating a series of steps, with each step building on the previous results. 2.13 **Fractals** are formed by recursion. 2.14 The beginning point of a fractal, the seed, is called **Stage 0**; with each recursion, the resulting iteration is labeled with a higher number, e.g. .the first iteration is Stage 1, the second is Stage 2, the third is Stage 3. 2.15 **Geometric fractals** are the lines, shapes and patterns that can be generated by iterations that follow specific rules. The patterns have identical self-similarity and are exact scaled copies of the original. There may be infinite stages. Examples are Cantor Dust, the Koch snowflake, and the Sierpinski Triangle	2.21 **Recursion** uses part of the output as input for the next iteration. 2.22 **Statistical or Stochastic fractals** are the patterns produced in nature where the iterative process is impacted by random accidental parameters. The patterns are statistically self-similar; that is, the parts have the same statistical properties as the original. Examples are human blood vessels and neurons, the branching of trees, and the growth of bacterial colonies	2.31 **Fractal patterns** are formed through geometric, algebraic, and natural processes. 2. 32 **Geometric Fractals** are the lines, shapes and patterns that can be generated by iterated function systems following specific rules. The patterns have identical self-similarity and are exact scaled copies of the original. There may be infinite stages. Examples are Cantor Dust, the Koch snowflake, and the Sierpinski Triangle. 2. 33 **Algebraic or Escape Time fractals** are the patterns generated by nonlinear iterative processes in which the range of input values is controlled. By adjusting a few input values, changes can occur in the resulting fractal. The patterns are self-similar, but are not exact scaled copies of the original. There may be infinite stages. Examples are the Mandelbrot and Julia sets. 2. 34 **Statistical or Stochastic fractals** are the spatial and temporal patterns produced in nature where the iterative process is impacted by random accidental parameters. The patterns are statistically self-similar; that is, the parts have the same statistical properties as the original. Self-similarity holds over a limited range of scales. Examples are human blood vessels and neurons, the branching of trees, and the growth of bacterial colonies.

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Scale

3. Fractals look the same at any scale.

Beginner	Intermediate	Advanced
3.11 To **Zoom** means to look closer or further away, that is, to change scale. Fractals are geometric shapes or patterns that have about the same form whether they are examined from close up or far away. 3.12 A mathematical fractal is **self-similar** at all stages. Examples of mathematical fractals are the Sierpinski Triangle, Koch Snowflake, Mandelbrot set. 3.13 **Natural fractals** are approximately self-similar at each repeated stage. Because they are influenced by random events in nature, they are not exactly self-similar. Examples of natural fractals include trees, the human respiratory, circulatory and nervous systems, and erosion patterns.	3.21 **Scale factor** is a constant equal to the ratio of two proportional lengths of corresponding similar figures. It is also called the constant of proportionality. The ratio of the areas of two similar figures is the square of the scale factor and the ratio of the volumes is the cube of the scale factor. 3.22 A fractal is **scale invariant –** it looks the same at any scale. This is a form of symmetry	3.31 Studying the **perimeter-area** ratio of a closed region for different stages may help to identify a fractal pattern because the length of fractal perimeters increases without bound as the scale at which they are measured goes to 0.

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Dimensions

4. Fractals have unique dimensions (roughness) that can be mathematically described.

Beginner	Intermediate	Advanced
4.11 A fractal has **roughness** that differs from classical geometric shapes. 4.12 Throughout the **stages** of a mathematical fractal, the properties of self-similarity and dimension remain constant. 4.13 **Fractal dimension**, a measure of roughness, is based on the number of new pieces and the relative size of these pieces in each new iteration of a self-similar pattern.	4.21 **Smooth** mathematical or theoretical shapes have classical Euclidean integral dimensions: 0 for a point, 1 for a line, 2 for a plane, and 3 for a solid. 4.22 Fractals have **rough** surfaces or edges that cannot be described by Euclidean shapes and formulas. Fractals are in-between classic Euclidean shapes and may be described by non-integral values. For example, a von Koch snowflake has a fractal dimension between 1 and 2 and a Menger sponge has a fractal dimension between 2 and 3. 4.23 **Fractal dimension** is a measure of irregularity or roughness that describes how completely a fractal appears to fill a line, a plane or space. 4.24 **Box-counting** using different scaled grids is a quantitative method of determining the fractal dimension (roughness) of statistically self-similar objects. A curve is covered with a grid of squares and the number of squares that the curve passes through is counted. The process is repeated with grids of smaller and smaller squares. As the grids become smaller or finer, the measured "size" of the fractal increases. The rate at which the proportion of filled squares increases is a measure of the fractal dimension.	4.31 The **Hausdorff Dimension** is used frequently to calculate fractal dimension. The fractal dimension is the slope of the log-log graph, where the independent variable is **M,** the magnification, and dependent variable is **N**, the number of new pieces. M is the reciprocal of r, the scale factor ($M = 1/r$). 4.32 The Hausdorff Dimension equation is a power law and can be applied directly when all of the pieces are scaled by the same amount. The **Moran equation** may be used to calculate fractal dimension when the different pieces are scaled by different amounts. 4.33 A fractal has dimension **greater than** its topographical dimension.

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Computer Technology

5. Computer technology has enabled centuries of mathematical research to be understood and expanded to form the discipline of fractal geometry.

Beginner	Intermediate	Advanced
5.11 Cantor, Hilbert, Sierpinski and von Koch worked with simple geometrical fractals based on points, lines and polygons. 5.12 Fractal geometry was developed in the late 20th century by Benoit Mandelbrot. 5.13 Clark Kimberling is credited with defining fractal sequences in 1995.	5.21 Iteration and recursion are valid problem solving techniques. 5. 22 Iterated Function Systems enable geometrical and algebraic explorations. 5.23 Fractal software allows mathematical exploration of music, art and architecture.	5. 31 Gaston Julia and Benoit Mandelbrot are mathematicians who contributed to the understanding of transformations or mappings of the complex plane and their visual representations. 5.32 The Mandelbrot set is a graph of all the Julia sets. The Mandelbrot set exists in parameter space while the Julia set exists in dynamic space. 5.33 Fractal software can perform electronic box-counts using digital images to determine fractal dimension.

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Model Nature

6. Fractal patterns are used to model nature in creating art, music, architecture and new processes, tools, and products that benefit society.

Beginner	Intermediate	Advanced
6.11 By applying the principles of fractal geometry, scientists are developing new ways to describe, interpret, predict, and model the natural world. 6.12 **L-systems** (Lindenmayer-systems) can model both geometric and stochastic fractals (Koch, Sierpinski, ferns, trees, and fractal sequences). 6.13 **Self-similar** patterns are found in nature, art and architecture. 6.14 Fractal patterns have been identified in artworks around the world: music compositions by Bach and Beethoven, architecture of African villages, and rugs from the Middle East. 6.15 Practical applications of fractals include military camouflage, realistic-looking virtual landscapes, and wallpaper to relax the mind. 6.16 Using a fractal pattern, one can compose a melody, creating spatial or temporal self-similarity.	6.21 Fractal geometry allows quantitative analysis of previously unquantifiable phenomena in a variety of fields. 6.22 By string re-writing, **L-Systems** can model natural fractals by generating branching patterns that mimic plants and living organisms. 6.23 The **random walker** model can describe the reaction-diffusion process involved in a variety of processes, e.g., termite paths and DNA nucleotide patterns. 6.24 Humans are engaged in smoothing the environment (deforestation, lubricants), yet tend to be more attracted to fractal designs, perhaps because the designs reflect nature. 6.25 Increasing the surface area of a given volume increases the exchange profile, e.g., human lungs, plant roots, and cell phone antennas for better signal reception.	6.31 **Diffusion-Limited Aggregation** is a fractal model for growth formations of sponge and coral, dendrites, viscous fingering, lightning and percolation. 6.32 **Cellular automata** are used to model physical, chemical, biological, and social interactions because cellular automata can produce a considerable variety of behaviors, including some that appear organic, and also many that are fractal. 6.33 Mathematical power laws and fractal models are used to **describe** growth and patterns in nature. Examples include erosion, migration, interstellar cloud formations, forest fires and travertine formations in Yellowstone National Park. 6.34 Fractal models can help with **problem solving and troubleshooting**, e.g., cancer detection, tracking underground drainage systems, and rust analysis. 6.35 Fractal geometry helps to model natural phenomena to make **predictions** such as weather and storm tracking and financial analysis. 6.36 Fractals are **powerful tools** in physics, chemistry, computer science, geography, economics, and even music and art. Although fractals are not a panacea, fractals are new tools for probing the rugged surface of reality. 6.37 Fractal geometry can be used to create new **products**, e.g. image compression, HD video, and *GalaxyIt* search engine.

